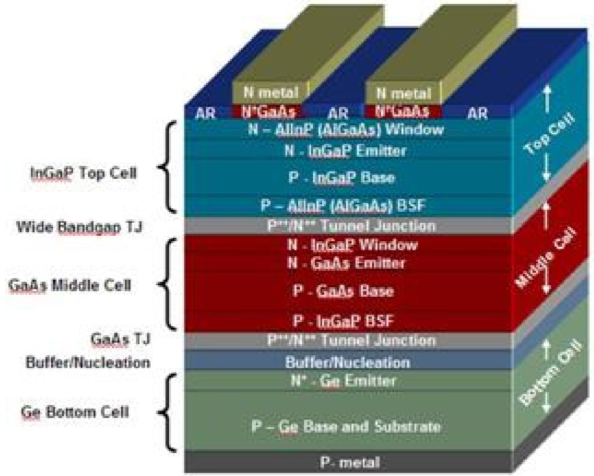
Lecture 9 – 13/11/2024

The *p-n* junction

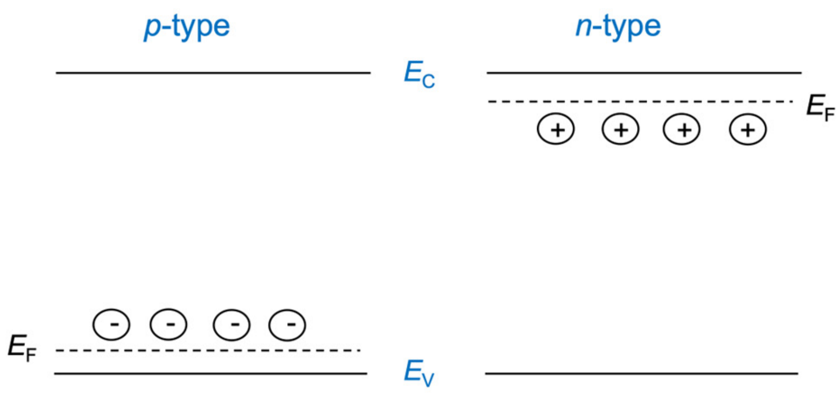
- With a concentration gradient
- Out of equilibrium
- *I-V* characteristic

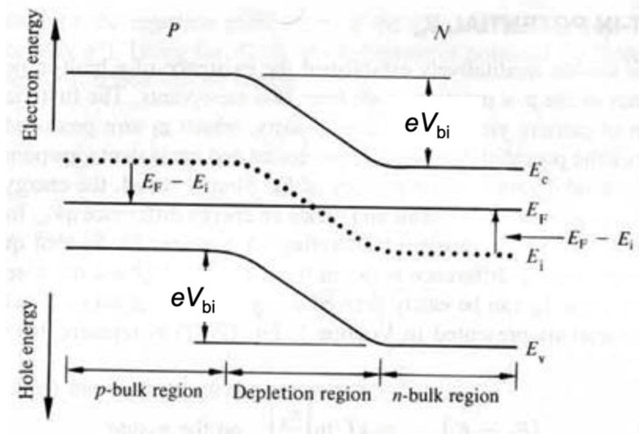


Summary Lecture 8

At thermal equilibrium

p-n junctions at thermal equilibrium





$(n \approx N_D$ *and* $p \approx N_A)$

- Concentration gradients $\Rightarrow$ diffusion currents
- Uncompensated ionized impurities $\Rightarrow$ built-in electric field $\Rightarrow$ drift currents

$$\mathbf{J}_{\mathbf{n,drift}} = \sigma_n \mathbf{E} = e\mu_n n\mathbf{E}$$

$$\mathbf{J}_{\mathbf{n,diff}} = eD_n\,\mathbf{grad}\,n$$

Einstein relation:

$$\boxed{\frac{D}{\mu} = \frac{k_B T}{e}}$$

At thermal equilibrium:

$$J_n = J_{n,\mathrm{drift}} + J_{n,\mathrm{diff}} = 0$$

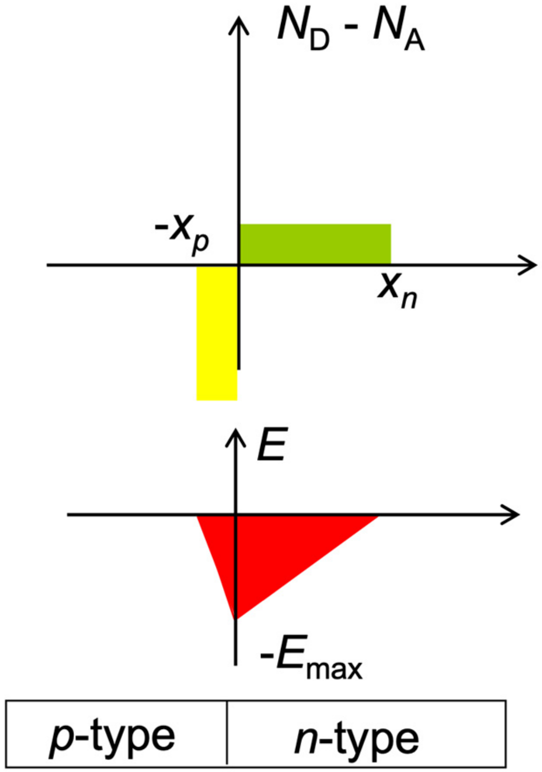
$$\boxed{J_n = e\mu_n nE + eD_n\frac{dn}{dx} = \mu_n n\frac{dE_F}{dx} = 0}$$

Built-in potential:

$eV_{bi} = (E_F - E_i)_n - (E_F - E_i)_p \equiv$ energy loss across the junction $= E_g - k_B T \ln\left(\dfrac{N_v N_c}{N_A N_D}\right)$

If all impurities are ionized

Summary Lecture 8



$$\boxed{\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_r \varepsilon_0}}$$ Electrostatic potential vs position

Charge neutrality: $x_p N_A = x_n N_D$

Space charge region extent: $W = x_p + x_n = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}$

Electric field in the space charge region

$$E(x) = -\frac{d\phi}{dx} = -e\frac{N_A(x + x_p)}{\varepsilon} \quad [-x_p, 0]$$

$$E(x) = -\frac{d\phi}{dx} = e\frac{N_D(x - x_n)}{\varepsilon} \quad [0, x_n]$$

$$V_{bi} = -\int_{-x_p}^{x_n} E(x)\,dx = \boxed{\frac{1}{2}E_{max}W} = e\frac{N_A x_p^2}{2\varepsilon} + e\frac{N_D x_n^2}{2\varepsilon}$$

Built-in potential:

$eV_{bi} = (E_F - E_i)_n - (E_F - E_i)_p \equiv$ energy loss across the junction

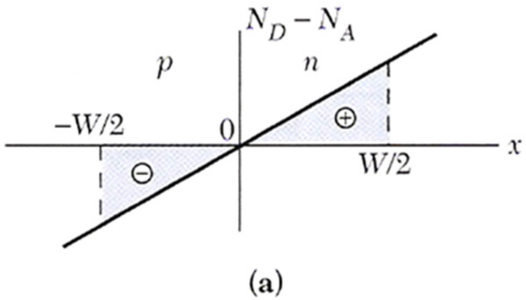
p-*n* junction with a gradient

- **The interface between the *n*-type and *p*-type doped layers is no longer abrupt: case of a linearly graded junction**

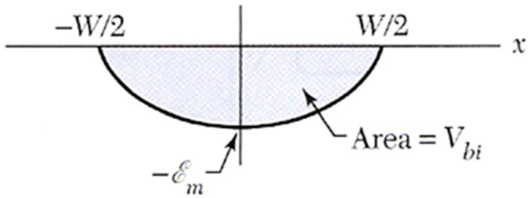
Density of charges given by $\rho = q\alpha x$ with α the impurity gradient (usually expressed in cm^{-4})

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon} = -\frac{q\alpha}{\varepsilon}x \qquad -\frac{W}{2} \le x \le \frac{W}{2}$$

Integrate between ±*W*/2 where the electric field becomes equal to zero (space charge boundaries)



Impurity distribution



Electric field distribution

p-n junction with a gradient

The built-in potential is then

$$V_{\mathrm{bi}} = \frac{e\alpha W^3}{12\varepsilon} \text{ and thus } W = \left(\frac{12\varepsilon V_{\mathrm{bi}}}{e\alpha}\right)^{1/3}$$

The built-in potential can also be expressed using a form similar to that of an abrupt junction:

$$V_{\mathrm{bi}} = \frac{k_{\mathrm{B}}T}{e}\ln\left(\frac{(\alpha W/2)(\alpha W/2)}{n_{\mathrm{i}}^2}\right) = \frac{2k_{\mathrm{B}}T}{e}\ln\left(\frac{\alpha W}{2n_{\mathrm{i}}}\right)$$

To be verified @ home!

Numerical techniques lead to:

$$V_{\mathrm{bi}} = \frac{2k_{\mathrm{B}}T}{3e}\ln\left(\frac{\alpha^2\varepsilon k_{\mathrm{B}}T}{8e^2 n_{\mathrm{i}}^3}\right)$$

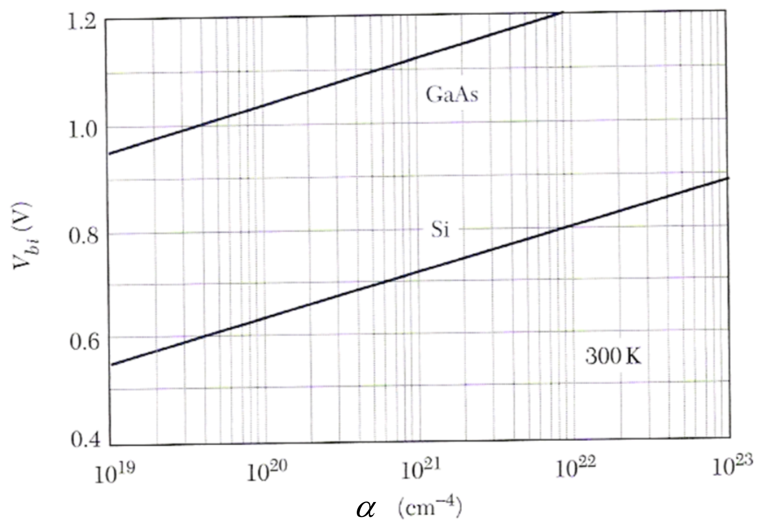
, for which V_{bi} is 0.05 to 0.1 V smaller than with the previous equation

Expression that only depends on known parameters

V_{bi} depends on the impurity gradient α

The smaller the gradient, the lower V_{bi}

W will vary as $(V_{\mathrm{bi}} - V)^{1/3}$ whereas in an abrupt junction it varies as $(V_{\mathrm{bi}} - V)^{1/2}$



Characteristics of the *p-n* junction at equilibrium

- Space charge
- Fixed charges due to depleted impurities
- No net current (drift and diffusion currents compensate themselves)

The *p-n* junction out of equilibrium

p-*n* junction out of equilibrium

Out of equilibrium means that the junction is polarized (external applied bias V)

Then, the expression of W as a function of V is still valid provided V_{bi} is replaced by $V_{bi} - V$

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right) V_{bi}}$$

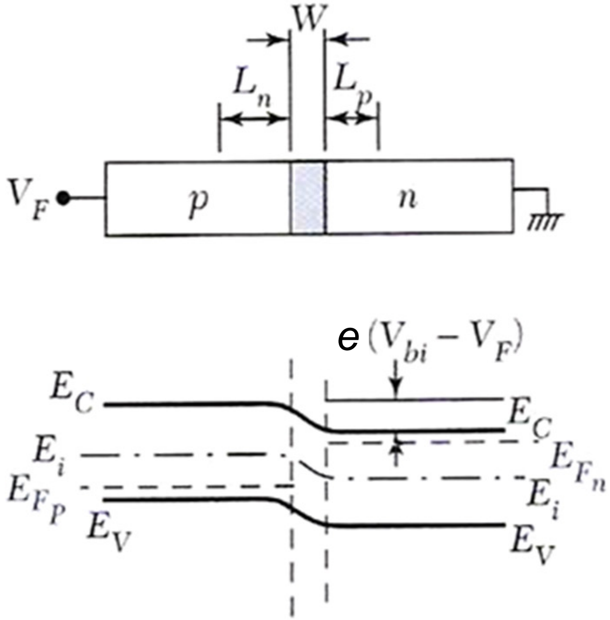
Abrupt *p*-*n* junction case!

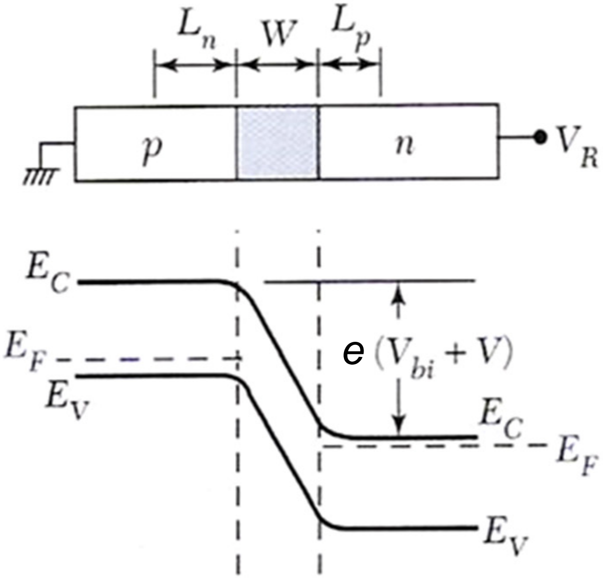
$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)\left(V_{bi} - V\right)}$$

p-*n* junction out of equilibrium

- Forward bias V_F: $V_{bi} - V_F$ thus $W \searrow$
- Reverse bias $-V_R$: $V_{bi} + V_R$ thus $W \nearrow$

$$W = \sqrt{\frac{2\varepsilon}{e}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V)}$$





E_{F_n} and E_{F_p} are the quasi-Fermi levels

Notion of quasi-Fermi levels

System driven out of equilibrium

- External generation process of electrons and holes $\Rightarrow$ generation rates G_n and G_p
- Recombination process with time constants τ_n and τ_p leading to steady-state electron and hole populations given by the equality between generation and recombination processes:

$$\boxed{G_n = \frac{\Delta n}{\tau_n} \text{ and } G_p = \frac{\Delta p}{\tau_p}}$$

See Lecture 7

It is assumed that the generated carriers (either by photon absorption, electrical injection, etc.) are present in densities exceeding by far thermal densities, i.e., $n, p >> n_0$ and p_0

Shockley introduced the concept which consists in considering that those free carrier populations can still be described by so-called *quasi-Fermi levels*

Notion of quasi-Fermi levels

If the system is non-degenerate, we get:

$$E_{F_n} = E_C - k_B T \ln\left(\frac{N_C}{n}\right) \text{ with } n = n_0 + G_n\tau_n \leftarrow \Delta n$$

$$E_{F_p} = E_V + k_B T \ln\left(\frac{N_V}{p}\right) \text{ with } p = p_0 + G_p\tau_p \leftarrow \Delta p$$

If the system is degenerate, we get:

$$E_{F_n} = E_C + \frac{\hbar^2}{2m_C^*}\left(3\pi^2 n\right)^{2/3}$$

$$E_{F_p} = E_V - \frac{\hbar^2}{2m_V^*}\left(3\pi^2 p\right)^{2/3}$$

As it is assumed that $G_n\tau_n >> n_0$ and $G_p\tau_p >> p_0$, quasi-Fermi levels do not coincide with each other and are even repelled from the equilibrium Fermi level position E_F by the amount:

$$\left(E_{F_n} - E_F\right) - \left(E_{F_p} - E_F\right) \approx k_B T \ln\left[\left(\frac{G_n\tau_n}{n_0}\right) / \left(\frac{G_p\tau_p}{p_0}\right)\right]$$

Non-degenerate semiconductor case

p-*n* junction out of equilibrium

Assumptions made for the derivation of the ideal current-voltage characteristic

1. Abrupt depletion layer, outside the depletion region the semiconductor is assumed to be neutral

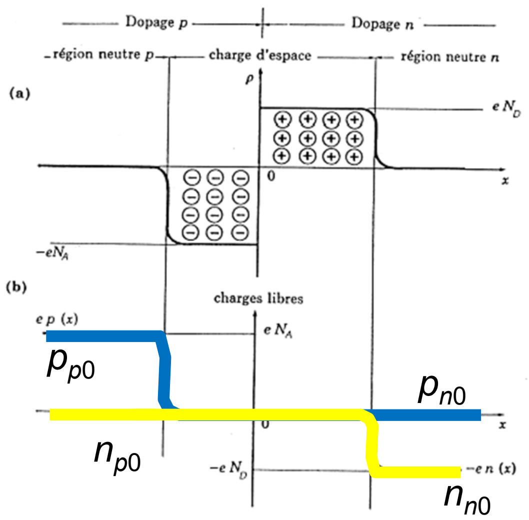
2. Carrier densities at the boundaries are related to the electrostatic potential difference across the junction

3. System operated under low-injection condition, i.e., injected minority carrier densities are small vs. majority carrier densities (i.e., $n_n \approx n_{n0}$ and $p_p \approx p_{p0}$) ⚠ ⇒ We do not rely on the conditions of slide #11!

4. Neither generation nor recombination current exists in the depletion region, and the electron and hole currents are constant throughout the depletion region (i.e., all currents come from the neutral regions)

p-n junction out of equilibrium

$V = 0$



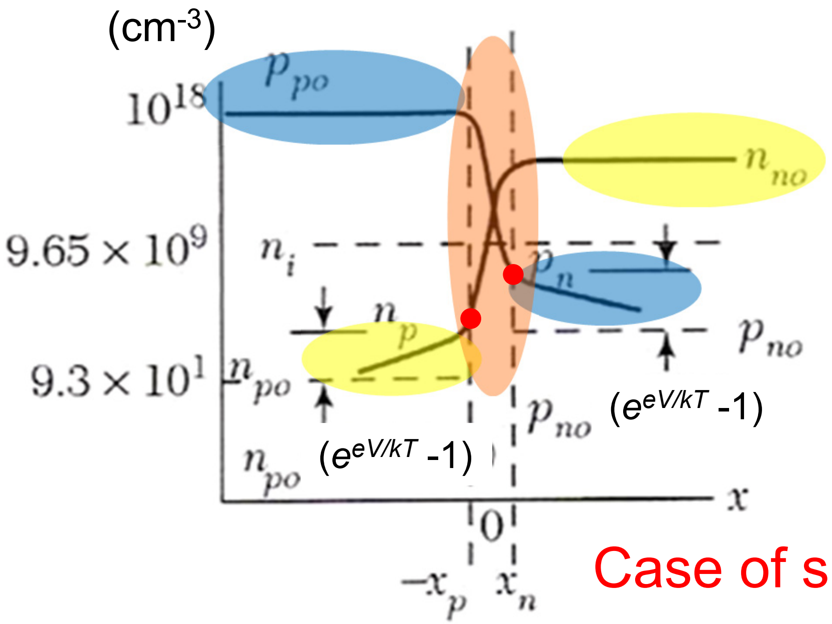
n-type:

$n_{n0} = N_D$ and $p_{n0} = n_i^2/N_D$

p-type:

$p_{p0} = N_A$ and $n_{p0} = n_i^2/N_A$

$V > 0$



Case of silicon

n_p and p_n are the concentrations of electrons and holes, respectively, at the space charge boundaries

I-V characteristic

At equilibrium in the *n*-type and *p*-type doped layers, we have:

$$n = N_c e^{-(E_C - E_F)/k_B T}$$

$$p = N_v e^{-(E_F - E_V)/k_B T}$$

$$eV_{bi} = E_g - k_B T \ln\left(\frac{N_v N_c}{N_A N_D}\right) = E_g - k_B T \ln\left(\frac{np}{N_A N_D} e^{(E_C - E_F)/k_B T} e^{(E_F - E_V)/k_B T}\right) = k_B T \ln\left(\frac{N_A N_D}{np}\right)$$

$$= k_B T \ln\left(\frac{n_{n0} p_{p0}}{n_i^2}\right) = k_B T \ln\left(\frac{n_{n0}}{n_{p0}}\right) \quad \text{with } n_{p0} p_{p0} = n_i^2 \qquad n_{n0} = N_D \text{ and } p_{p0} = N_A$$

We can deduce

$$n_{p0} = n_{n0} \exp(-eV_{bi}/k_B T)$$

and

$$n_{n0} = n_{p0} \exp(eV_{bi}/k_B T)$$

At thermal equilibrium

Electron and hole concentrations at both sides of the space charge region are a function of V_{bi}

I-V characteristic

$n_{n0} = n_{p0} \exp(eV_{bi}/k_B T)$
$p_{p0} = p_{n0} \exp(eV_{bi}/k_B T)$

These relations are still valid when the junction is biased

<u>Polarization (forward: $V > 0$, reverse: $V < 0$)</u>

$$\boldsymbol{n_n = n_p \exp[e(V_{bi}-V)/k_B T]}$$

Assumption #2 of slide #12

with n_n and n_p the **concentrations at the space charge region boundaries**

We consider the case of the low injection regime, i.e., $n_n \approx n_{n0}$

$$n_{n0} \approx n_n \Rightarrow n_{p0}\exp(eV_{bi}/k_B T) \approx n_p \exp[e(V_{bi}-V)/k_B T]$$

and finally $\boldsymbol{n_p \approx n_{p0} \exp(eV/k_B T)}$

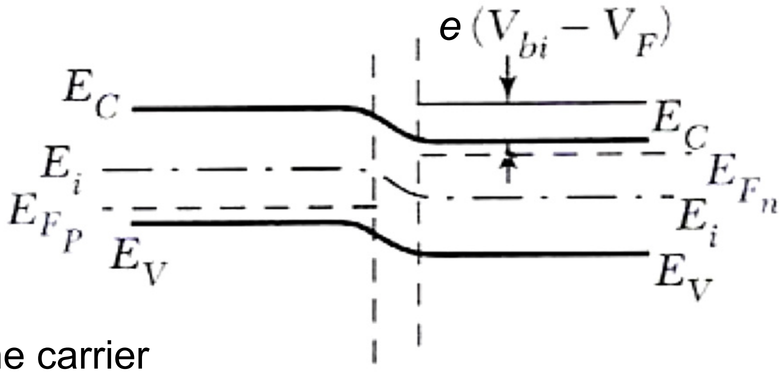
Excess or deficit of free carriers vs thermal equilibrium

or $\quad n_p - n_{p0} \approx n_{p0}[\exp(eV/k_B T)-1]$ on the p-type side at $x = -x_p$

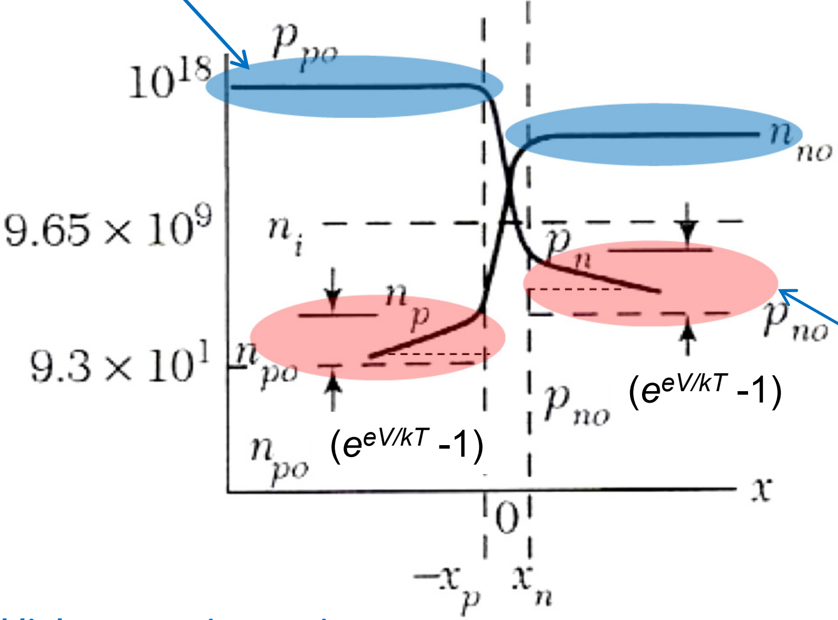
Similarly, we have $\quad p_n - p_{n0} = p_{n0}[\exp(eV/k_B T)-1]$ on the n-type side at $x = x_n$

Shockley's relations

I-V characteristic



We have expressed the carrier concentrations at both sides of the space charge region



We have to describe the evolution of the minority carrier concentration far from the junction (when going back to equilibrium)

I-*V* characteristic

Minority carrier concentrations in the neutral *n*- and *p*-type doped layers

The carriers come from the *n*- and *p*-type doped layers

Currents:

Rate equation: $$\frac{dn}{dt} = \frac{1}{e}\nabla \mathbf{J_n} + \left(G_n - R_n\right)$$

G_n: *generation rate*

R_n: *recombination rate*

with $$\mathbf{J_n} = e\left(\mu_n n\mathbf{E} + D_n \vec{\nabla} n\right)$$

Reminder

$G_n - R_n$ is given by:

$$G_n - R_n = -\frac{n_p - n_{p_0}}{\tau_n}$$

Out of equilibrium population

Lifetime (relaxation time)

Cf. Lecture 7

I-V characteristic

Steady-state regime (with no electric field in the neutral part of the *p*- and *n*-type doped regions)

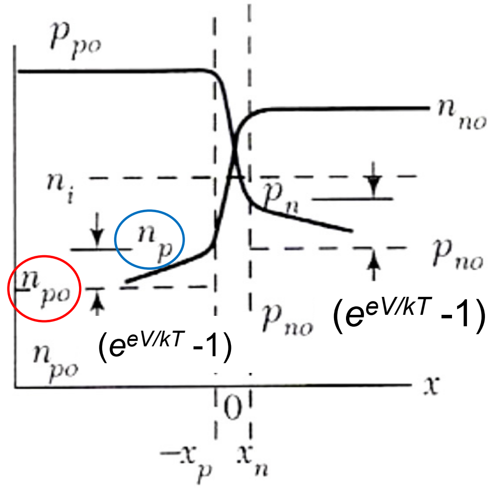
$$\frac{dn}{dt} = \frac{1}{e}\frac{dJ_n}{dx} + (G_n - R_n)$$

$$J_n = e(\cancel{\mu_n n E} + D_n\, dn/dx)$$

$$\frac{dn}{dt} = 0 = D_n \frac{d^2 n_p}{dx^2} + (G_n - R_n)$$

$$= D_n \frac{d^2 n_p}{dx^2} - \frac{(n_p - n_{p_0})}{\tau_n}$$

$$\frac{d^2 n_p}{dx^2} - \left(\frac{n_p - n_{p0}}{D_n \tau_n}\right) = 0$$



I-V characteristic

$$\frac{d^2 n_p}{dx^2} - \left(\frac{n_p - n_{p0}}{D_n \tau_n}\right) = 0$$

Integrate with $n_p = n_{p0}\exp(eV/k_BT)$ at $x = -x_p$ and $n_p = n_{p0}$ at $x = -\infty$

Importance of boundary conditions!

$\Rightarrow n_p - n_{p0} = n_{p0}[\exp(eV/k_BT) - 1]\exp[(x+x_p)/L_n]$ with $\boxed{L_n = \sqrt{D_n \tau_n}}$

⚠ $n_p = n_p(x)$

Shockley's relation, cf. slide #15

L_n is the diffusion length of electrons in the p-type doped layer

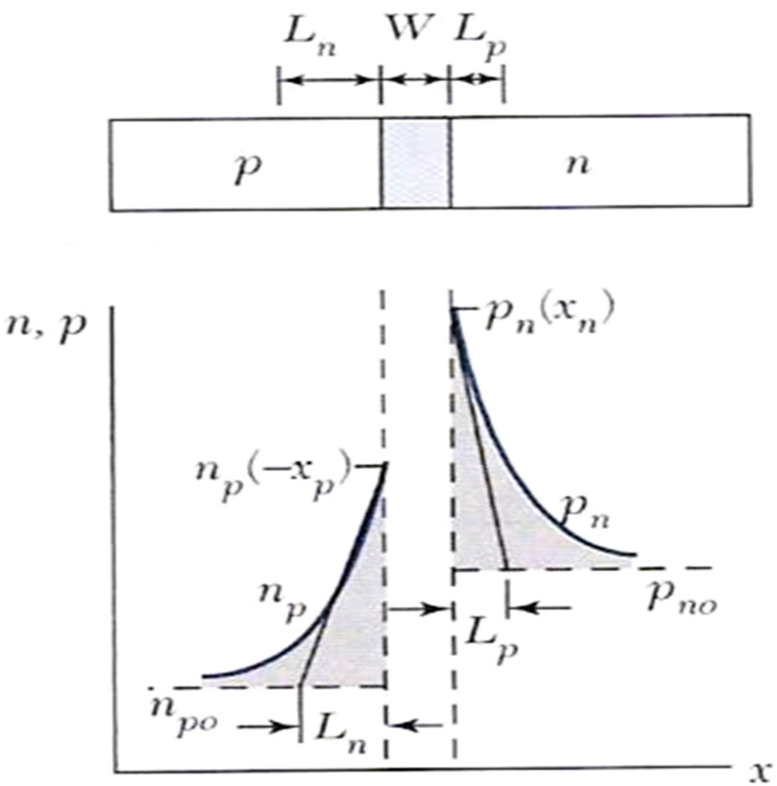
The electron current density at $-x_p$ is then given by

$$J_n(-x_p) = eD_n \left.\frac{dn_p}{dx}\right|_{-x_p} = e\frac{D_n n_{p0}}{L_n}\left(e^{eV/k_BT} - 1\right)$$

and similarly the hole current density at x_n is equal to

$$J_p(x_n) = -eD_p \left.\frac{dp_n}{dx}\right|_{x_n} = e\frac{D_p p_{n0}}{L_p}\left(e^{eV/k_BT} - 1\right)$$

I-V characteristic



The minority carriers recombine with the majority carriers while going away from the space charge region

Ideal I-V characteristic

The total current is:

$$J = J_n(-x_p) + J_p(x_n)$$
$$= \frac{eD_n n_{p0}}{L_n}(e^{eV/k_BT} - 1) + \frac{eD_p p_{n0}}{L_p}(e^{eV/k_BT} - 1)$$

$$\boxed{J = J_s(e^{eV/k_BT} - 1)} \quad \textit{with} \quad J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$ ***Saturation current density***

$n_{p0} = n_i^2/N_A$ and $p_{n0} = n_i^2/N_D$

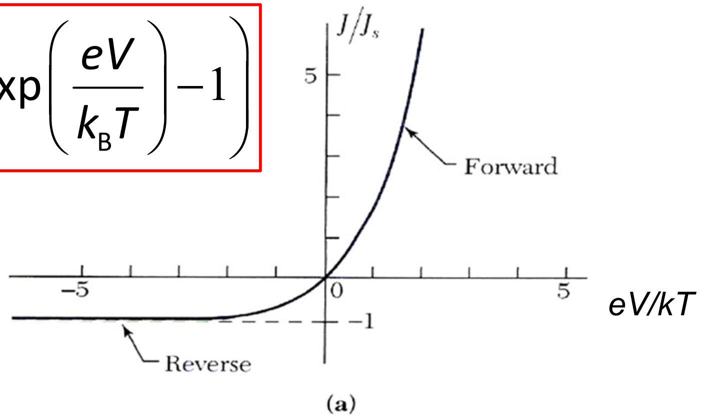
$$\boxed{J_s = \frac{eD_n n_i^2}{N_A L_n} + \frac{eD_p n_i^2}{N_D L_p}}$$

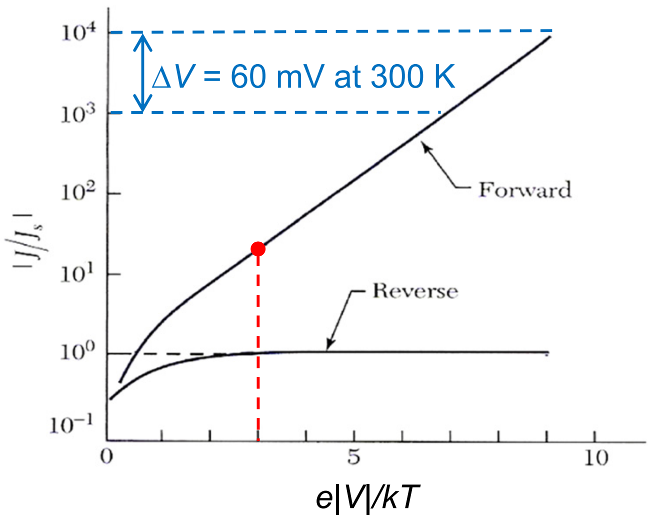
with $n_i^2 = N_c N_v e^{-\left(\frac{E_c - E_v}{k_B T}\right)} = N_c N_v e^{-\left(\frac{E_g}{k_B T}\right)}$

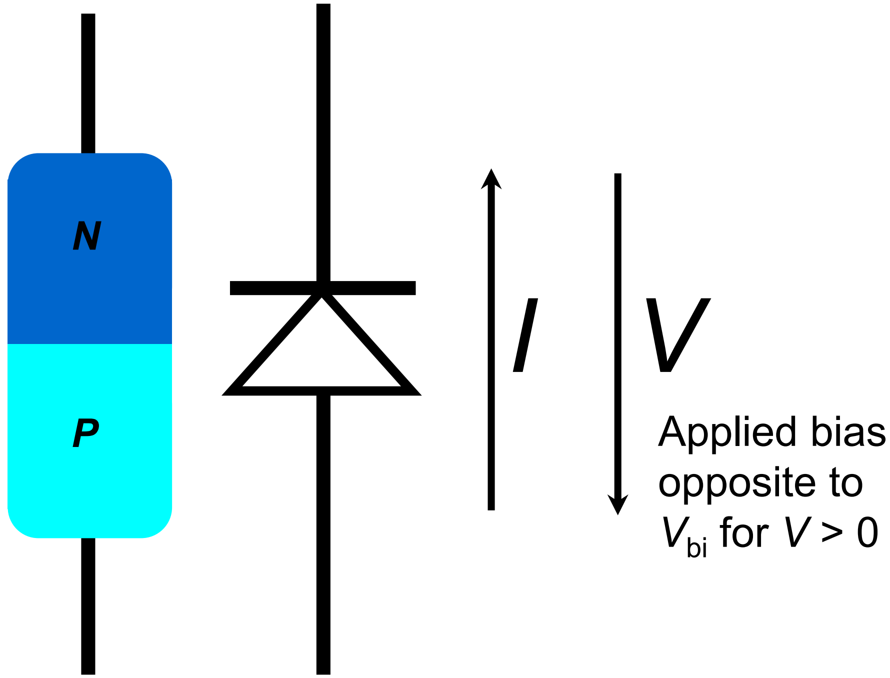
- For $V \geq 3k_BT/e$, the rate of current increase is constant
- At 300 K, for every decade change of current, the voltage change for an ideal diode is 60 mV (= $2.3k_BT/e$)
  Cf. remark of slide #8 of Lecture 8
- In the reverse direction, the current density saturates at $-J_s$

Ideal *I*-*V* characteristic

$$J(V) = J_S\left(\exp\left(\frac{eV}{k_B T}\right) - 1\right)$$







V = 0, no current (equilibrium)

V < 0, *J* tends to – J_s (hence the denomination ***saturation current density***)

V > 0, *J* rapidly increases

Ideal *I*-*V* characteristic

J_s in a silicon-based *p*-*n* junction

$N_A = 5 \times 10^{16}$ cm^{-3}, $N_D = 1 \times 10^{16}$ cm^{-3}, $n_i \approx 10^{10}$ cm^{-3}
$D_n = 21$ cm²/s, $D_p = 10$ cm²/s, $\tau_n = \tau_p = 5 \times 10^{-7}$ s

$J_s = 8.6 \times 10$ pA/cm^2

The typical size of a device is a few squared microns in cross section, which leads to I_s values on the order of a few fA

There is "no current" at reverse bias

In contrast, at $V = 1$ V (forward bias), $J = 2 \times 10^2$ A/cm^2